Math 111

Notes by May on Conic Section

Spring Semester 2019

The absolute value of a number is the non-negative value of it, regardless the sign.
But it could also be thought of as the number's distance from zero.

Exg- $|-2| = 2$ distance from 0 to $-2 = 2$

That is why we use it to refer to the distance between two points.

Exg- $|AB| \rightarrow$ difference between A to B

↳ Length of the Line segment AB

say $A(5, 9)$ $B(4, 3)$

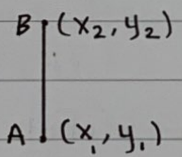
$$|AB| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(5-4)^2 + (9-3)^2}$$

* $|AB| = \sqrt{(\Delta x)^2 + (\Delta y)^2} \rightarrow$ when the line isn't vertical nor horizontal

absolute value

* $|AB| = |\Delta Y| \rightarrow$ Vertical Line

↳ value of x is the same



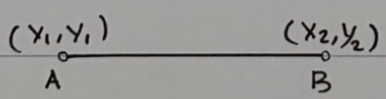
$\Delta x = x_1 - x_2 = 0$

$\therefore x_1 = x_2$

take absolute value

* $|AB| = |\Delta x| \rightarrow$ Horizontal Line

↳ values of y are the same



$\Delta y = y_1 - y_2 = 0$

$y_1 = y_2$

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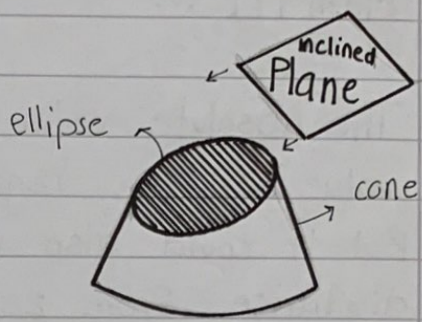
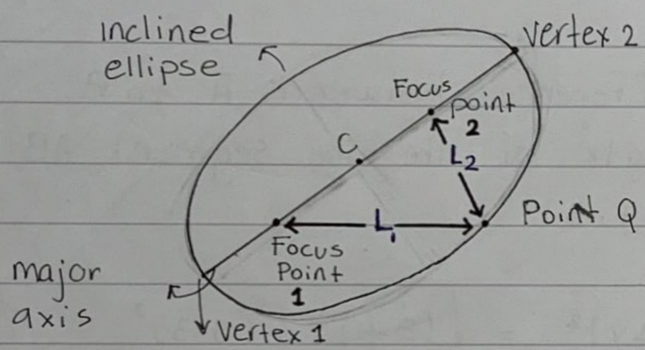
ELLIPSE

lecture 1

An ellipse is a curve in a plane surrounding two focal points F_1, F_2 such that the sum of the distances to the focal points is constant (k) for every point on the curve.



an ellipse is obtained from the intersection of a cone with an inclined plane.



* The sum of L_1 & L_2 is equal to a constant amount call it k

when sketching :

Fix two points on a horizontal or a vertical line Say F_1 & F_2 The points are the Foci. Surround them by the curve (ellipse)

* The Foci lie inside the ellipse

Note that k has to be bigger than $|F_1F_2|$

* Q is a point on the ellipse

*

$$|QF_1| + |QF_2| = K$$

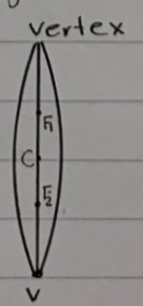
* K is the major axis

* Ex:-

$F_1(1, 7)$

$F_2(1, -3)$

major axis (vertical)



$|F_1F_2| = |\Delta Y| = |10| = 10$

k has to be greater than 10

$c = \text{average} = \frac{7+(-3)}{2} = 2$

Observations :

1. The Foci lie inside the ellipse
2. Foci always lie on the major axis
3. The major axis = K
4. The center is the mid point of the line V_1V_2

$V_1V_2 \Rightarrow$ major axis

5. c is calculated from the average of the vertices

note : The reason why we drew a vertical ellipse is because you can notice from $F_1(1,7)$ & $F_2(1,-3)$ they are on a vertical line & the Foci lie on the major axis

$\therefore$ major axis = vertical line

Lecture 2

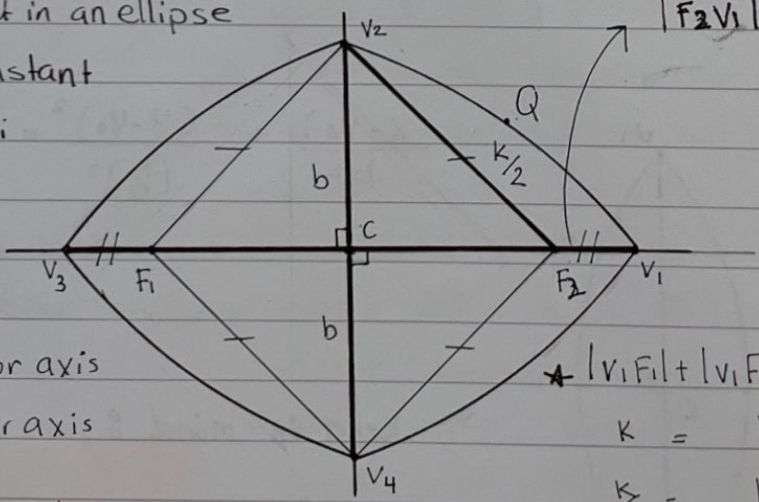
Knowing that in an ellipse

$K \to$ is constant

$F_1 F_2 \to$ Foci

$K > |F_1F_2|$

★ **note :** $|F_2V_1| = |V_3F_1|$



$\overline{V_1V_3}$ = major axis

$\overline{V_2V_4}$ = minor axis

★ $|QF_1| + |QF_2| = K$

↳ can be used withe the vertices because each vertex is a point on the ellipse.

so ★ $|V_1F_1| + |V_1F_2| = K$

★ $|V_1F_1| + |V_1F_2| = K = |V_1V_3|$

$K = |\text{major axis}|$

$\frac{K}{2} = |\frac{1}{2}\text{ major axis}|$

$\frac{K}{2} = |CV_1| = |CV_3|$

★ note $K = |V_2F_1| + |V_2F_2|$

$|V_2F_1| = |V_2F_2|$

$K = 2|V_2F_1|$

$\frac{K}{2} = |V_2F_1|$

applies on V_4 as well

From the drawing:

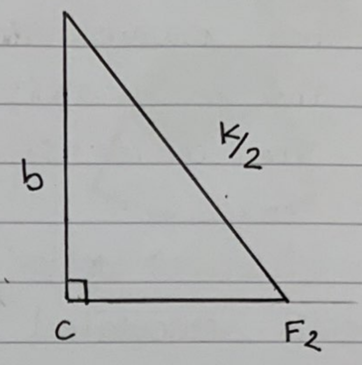
we conclude

$$\left(\frac{K}{2}\right)^2 = b^2 + \left(|CF_2|\right)^2$$

$$b^2 = \left(\frac{K}{2}\right)^2 - \left(|CF_2|\right)^2$$

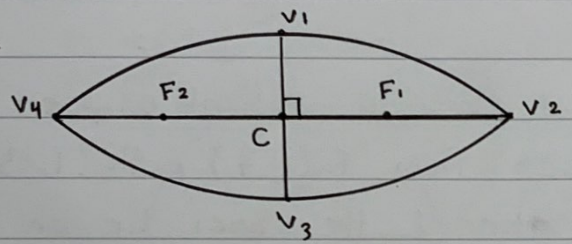
$2b \rightarrow$ Minor axis

minor axis ↑



note that:

- ★



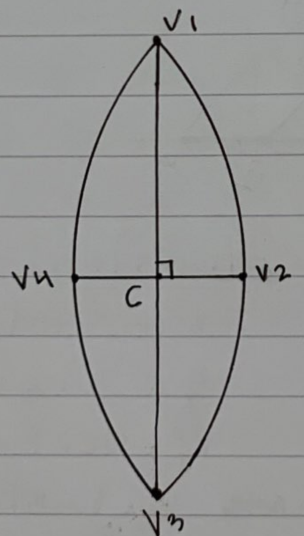
$C(x_0, y_0)$

$$\frac{(x - x_0)^2}{\left(\frac{K}{2}\right)^2} + \frac{(y - y_0)^2}{b^2} = 1$$

The $\left(\frac{K}{2}\right)^2$ is denominator of the x's

x axis → horizontal ellipse ⟷ x

- ★



$$\frac{(x - x_0)}{b^2} + \frac{(y - y_0)^2}{\left(\frac{K}{2}\right)^2} = 1$$

The key is denominator of y's

y axis → vertical ellipse ↕ y

Keep in mind:

$$\left(\frac{K}{2}\right)^2 > b^2$$

↳ greater value

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Question 1: Assume (2,5) & (2,9) are foci of an ellipse with a constant $k = 5$

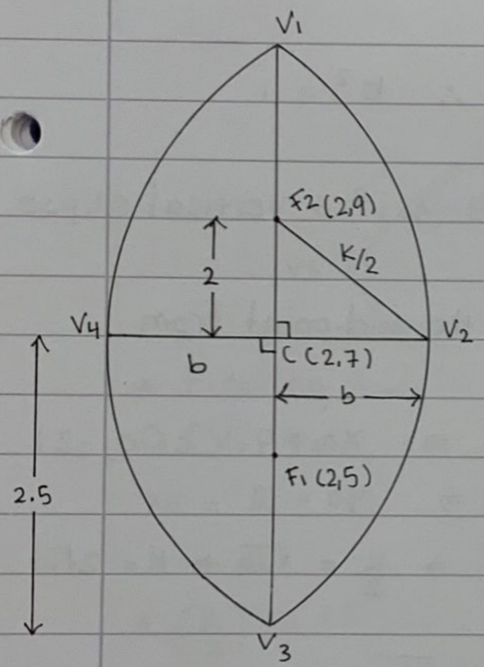
(1) Find all vertices

(2) Roughly sketch the figure

(3) Find the length of Major & Minor axis

(4) Write down the equation of such an ellipse

* step 1: roughly sketch → [Because the foci lie on a vertical line (The x is the same)
↳ so the ellipse is vertical]



* say $F_1(2,5)$ & $F_2(2,9)$
* Find c → by the average of y

$c(x_0, y_0)$ $x_0 = 2$ → on a vertical line

$y_0 = \frac{9+5}{2} = \frac{14}{2} = 7$ $c(2,7)$

* To find the vertical vertices

calculate $k/2$ → $k = 5$ → given ⇒ $\frac{k}{2} = 2.5$

$V_1 \Rightarrow (2, 7+2.5) \Rightarrow V_1(2, 9.5)$ moving to the right

$V_3 \Rightarrow (2, 7-2.5) \Rightarrow V_3(2, 4.5)$

↳ on the vertical axis x → the same

* To find the horizontal vertices

calculate b

step 1: find $|cF_2|$

$|cF_2| = \cancel{\Delta x} = \Delta Y = |7-9| = 2$

step 2: Draw triangle

$(\frac{k}{2})^2 = b^2 + |CF_2|^2$

$(2.5)^2 = b^2 + 2^2$

$b^2 = 2.25 \Rightarrow b = 1.5$

$V_2 \Rightarrow (2+1.5, 7) \Rightarrow (3.5, 7)$

$V_4 \Rightarrow (2-1.5, 7) \Rightarrow (0.5, 7)$

↳ moving to left ↳ y doesn't change on a horizontal

* $|\text{Minor}| = 2b = 3$
* $|\text{Major}| = k = 5$
* equation

$$\frac{(x-x_0)^2}{b^2} + \frac{(y-y_0)^2}{(\frac{k}{2})^2} = 1$$

$$\frac{(x-2)}{2.25} + \frac{(y-7)^2}{6.25} = 1$$

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lecture 3

Question 2) $\frac{x^2}{1} + \frac{(y+2)^2}{10} = 1$

(1) sketch roughly ✓

(2) Find c ✓

(3) Vertices ? ✓

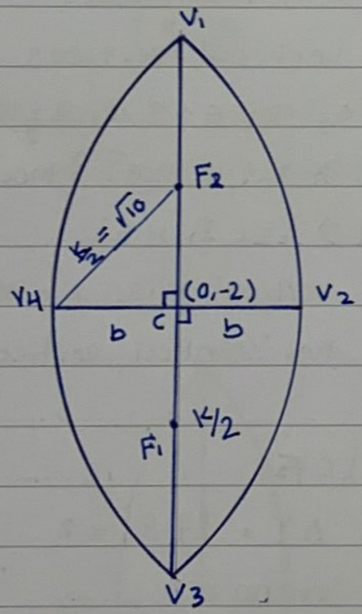
(4) Find the Foci ✓

(5) k = ? ✓

(6) |Minor| = ? |Major| = ? ✓

because $(\frac{k}{2})^2 > b^2$ $\therefore (\frac{k}{2})^2 = 10$ $\therefore b^2 = 1$

$(\frac{k}{2})^2$ is the denominator of $(y-y_0)^2 \rightarrow$ vertical ellipse



* Finding c the mid-point from the equation

$x - x_0 = x \Rightarrow x_0 = 0$ $c(0,-2)$

$y + 2 = y - y_0 \Rightarrow y_0 = -2$

* $(\frac{k}{2})^2 = 10 \Rightarrow \frac{k}{2} = \sqrt{10} \Rightarrow k = 2\sqrt{10}$

$|Major| = 2\sqrt{10}$

* $b^2 = 1$ $b = 1$ $|Minor| = 2(1) = 2$

* Finding horizontal vertices by b

$V_2\ (0+1, -2) \Rightarrow V_2(1,-2)$

$V_4\ (0-1, -2) \Rightarrow V_4(-1,-2)$

* Finding vertical vertices by k/2

$V_1\ (0, -2+\sqrt{10}) \Rightarrow V_1(0, 1.162)$

$V_3\ (0, -2-\sqrt{10}) \Rightarrow V_1(0, -5.16)$

↳ keep in this form

* Finding the foci by drawing the triangle

$(\frac{k}{2})^2 = b^2 + |cF_2|^2$

$10 = 1 + |cF_2|^2$

$|cF_2| = \sqrt{9} = 3$

$F_2(0, -2+3) \Rightarrow F_2(0,1)$

$F_1(0, -2-3) \Rightarrow F_1(0,-5)$

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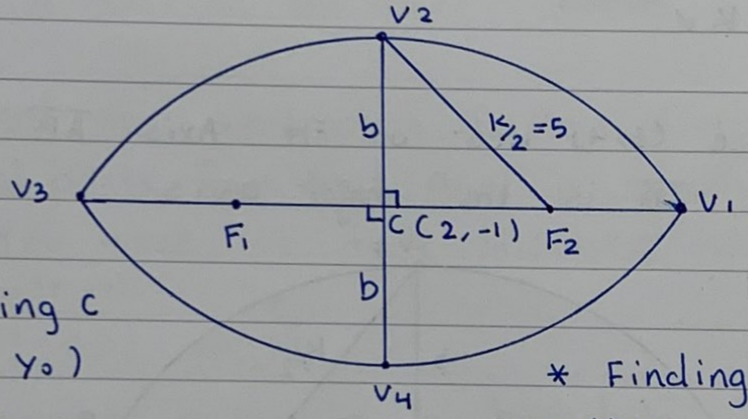
Question 3: $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1$

(1) sketch roughly
(2) Find c
(3) Find the Foci
(4) Find the vertices
(5) $|Major| = ?$ & $|Minor| = ?$

$25 > 9$ so $25 = \left(\frac{k}{2}\right)^2 \rightarrow \left(\frac{k}{2}\right)^2$ is the denominator of $(x-x_0)^2$
↳ horizontal ellipse.



* Finding c
$C(x_0, y_0)$
$x_0 = 2$
$y_0 = -1$
* $\left(\frac{k}{2}\right)^2 = 25$
$\frac{k}{2} = 5$
$k = 10$
$|Major| = 10$
* $b^2 = 9$
$b = \sqrt{9} = 3$
$|Minor| = (2)(3) = 6$
* Finding horizontal vertices by K/2
$V_1(2+5, -1) \Rightarrow V_1(7,-1)$
$V_3(2-5, -1) \Rightarrow V_3(-3,-1)$

* Finding vertical vertices by b
$b^2 = 9$ $b = \sqrt{9} = 3$
$V_2(2, 3-1) \Rightarrow V_2(2,2)$
$V_4(2, -1-3) \Rightarrow V_4(2,-4)$
* Finding the Foci by the triangle
$\left(\frac{k}{2}\right)^2 = b^2 + |CF_2|^2$

$25 = 9 + |CF_2|^2$
$|CF_2| = 4$
$F_2 \rightarrow (2+4, -1) \Rightarrow F_2(6,-1)$
$F_1 \rightarrow (2-4, -1) \Rightarrow F_1(-2,-1)$

note : $x^2 + \frac{(y-2)^2}{2} = 12$ can be turned into an ellipse's equation by dividing it by 12

$$\frac{x^2}{12} + \frac{(y-2)^2}{24} = 1$$ and then solve as usual

Question 4) $(10, 4)$ & $(-2, 4)$ are vertices of an ellipse & $(6, 4)$ is one of the foci Find:

Y Y Y $Y = 4$ on horizontal line

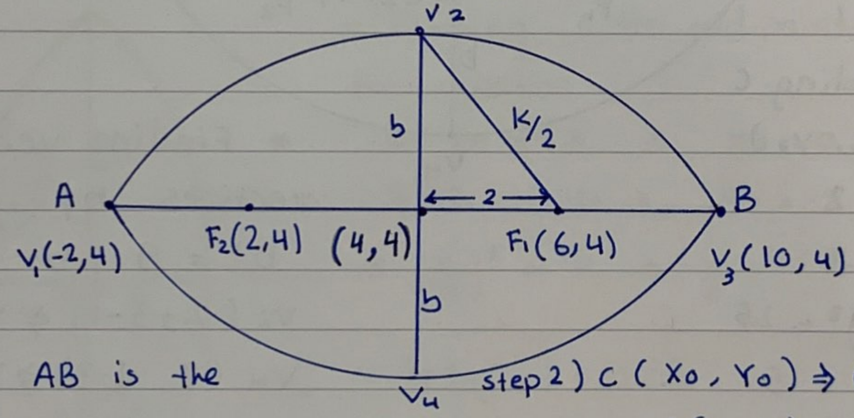
✓(1) The Second Foci

✓(2) sketch roughly

(3) Find all the remainig vertices

✓(4) Find K ✓

note : since $(6,4)$ lies on the Axis $\overline{AB}$ we conclude $\overline{AB}$ is the major axis



step 1) since AB is the major axis

$|AB| = K = |\Delta x| = |-2-10| = 12$

$K/2 = 6$

step 2) $c(x_0, y_0) \Rightarrow (-2+6, 4)$

$c(4,4)$

I calculated c to know where does $(6,4)$ lie ! and to find F_2.

step 3) $|CF_1| = |\Delta x| = 2$

note that $|cF_1| = |cF_2|$

$F_2(4-2, 4) \Rightarrow F_2(2, 4)$

step 4) Find b by the triangle

$(\frac{K}{2})^2 = b^2 + |cF_1|^2$

$(6)^2 = b^2 + (2)^2$

$b^2 = 32$

$b = 4\sqrt{2}$

step 5) $V_2(4, 4+4\sqrt{2})$

$V_4(4, 4-4\sqrt{2})$

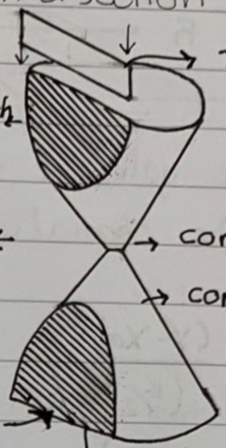
The equation is

$$\frac{(x-x_0)^2}{(K/2)^2} + \frac{(y-y_0)^2}{b^2} = 1$$

$$\frac{(x-4)^2}{36} + \frac{(y-4)^2}{32} = 1$$

Hyberbola

The pair of hyperbolas formed by the intersection of a plane with two equal cones on opposite of the same vertex.



When sketching:

Fix two points F_1 & F_2

The Foci (Focus points) on the major axis

choose $K > 0$ and $0 < K < |F_1F_2|$

choose a point on the figure say P.

note that the substraction of the distances to the focal points from that point p equals to K

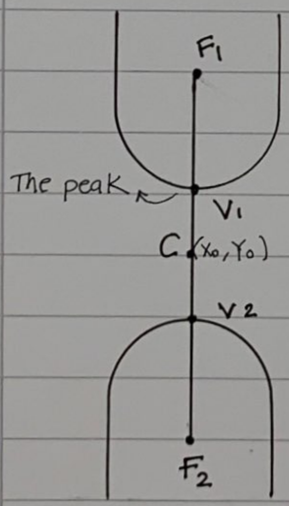
* You have to take absolute value → because $K > 0$

absolute value

$$||PF_1| - |PF_2|| = K$$

The solution to this is a hyperbola.

(1) The up-down hyperbola.



. The value of x in F_1, V_1, F_2 & V_2 is the same

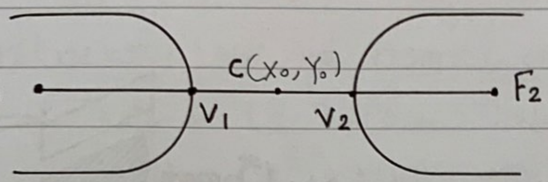
. The equation:

$$\frac{(y-y_0)^2}{(\frac{K}{2})^2} - \frac{(x-x_0)^2}{b^2} = 1$$

$(y-y_0)^2$ is the positive fraction → so the hyperbola is vertical (it has nothing to do with $(\frac{K}{2})^2$ & b^2)

The vertical line is the major axis.

(2) The left-right hyperbola.



- The value of Y in F_1, V_1, V_2 & F_2 is the same.
- The equation :-

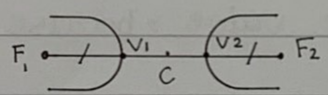
$$\frac{(x-x_0)^2}{(K/2)^2} - \frac{(y-y_0)^2}{b^2} = 1$$

. The $\frac{(x-x_0)^2}{(K/2)^2}$ is the positive fraction so the hyperbola is horizontal it has nothing to do with the denominator

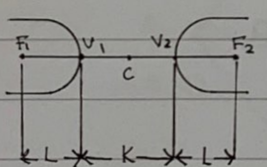
Remarks & observations :

(1) V_1 & V_2 are the vertices (peaks) of such hyperbola.

(2) $|F_1V_1| = |F_2V_2|$



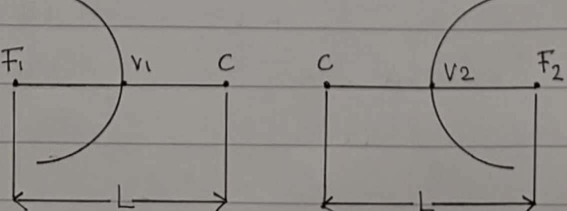
(3) $|V_1F_2| - |V_1F_1| = |V_1V_2| = K$



(4) c is the mid point
midpoint of $\overline{V_1V_2}$
mid point of $\overline{F_1F_2}$

(5) $|CV_1| = |CV_2| = K/2$

(6) $|CF_1| = |CF_2| = \sqrt{\left(\frac{K}{2}\right)^2 + b^2}$



$L = \sqrt{\left(\frac{K}{2}\right)^2 + b}$

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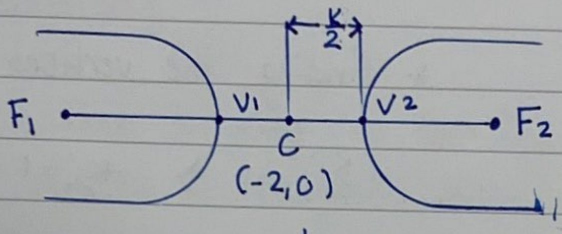
Question 1 : $\frac{(X+2)^2}{3} - \frac{y^2}{4} = 1$

↳ a hyperbola

The $(x-x_0)^2$ is the

Positive fraction → horizontal hyperbola



↳ The value of y for F_1 & V_1 & V_2 & F_2 is the & equals to (0)

• Finding c

from the equation

$c(x_0, Y_0)$

$X - X_0 = X+2$

$X_0 = -2$

$Y - Y_0 = Y$

$Y_0 = 0$

* $\left(\frac{K}{2}\right)^2$ is the denominator of the positive Fraction

$\left(\frac{K}{2}\right)^2 = 3 \Rightarrow \frac{K}{2} = \sqrt{3} \rightarrow K = 2\sqrt{3}$

* Finding the vertices by using $K/2$

$V_2(-2+\sqrt{3}, 0)$

$V_1(-2-\sqrt{3}, 0)$

↳ moving to left

* Finding the Foci by finding the Length of $\overline{CF_1}$

$|CF_1| = \sqrt{\left(\frac{K}{2}\right)^2 + b^2} = \sqrt{3+4}$ → From the equation

$|CF_1| = \sqrt{7}$

$F_1(-2-\sqrt{7}, 0)$

$F_2(-2+\sqrt{7}, 0)$

↳ To the right

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Question 2) $\dfrac{(y+3)^2}{(1)} - \dfrac{(x-1)^2}{15} = 1$ (y_0, x_0)

(1) sketch

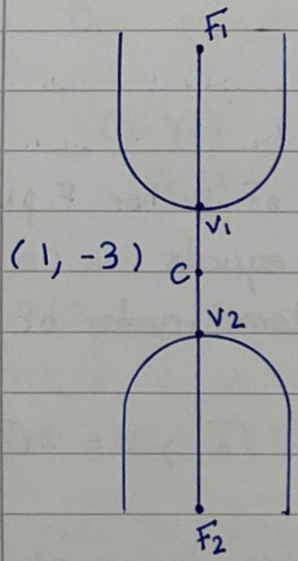
(2) Find F_1 & F_2

(3) vertices

(4) $K = ?$

* The $\dfrac{(y-y_0)^2}{(\frac{K}{2})^2}$ is the positive fraction

↳ vertical hyper bola



* Finding the vertices by using $K/2$

$(\frac{K}{2})^2 = 1 \Rightarrow K/2 = 1 \Rightarrow K = 2$

$V_1(1, -3+1) \Rightarrow V_1(1, -2)$

↓ From the midpoint C

$V_2(1, -3-1) \Rightarrow V_2(-1, -4)$

* Finding the Foci by the length of $\overline{CF_1}$ note: $\overline{CF_1} = \overline{CF_2}$

$|CF_1| = \sqrt{15+1} = \sqrt{16} = 4$

$F_1(1, -3+4) \Rightarrow (1,1)$ & $F_2(1, -3-4) \Rightarrow (1,-7)$

Question 3) $\dfrac{(x+5)^2}{4} - \dfrac{y^2}{5} = 1$ ($-(-5)$, -0)

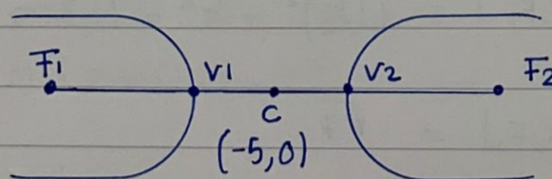
horizontal hyperbola

(1) sketch

(2) Find F_1 F_2

(3) Find V_1 V_2

(4) Find K



* Finding K & The vertices

$(\frac{K}{2})^2 = 4 \rightarrow \frac{K}{2} = 2 \Rightarrow K = 4$

$V_2(-5+2, 0) \Rightarrow V_2(-3, 0)$

$V_1(-5-2, 0) \Rightarrow V_2(-7, 0)$

* Finding the Foci

$|CF_1| = \sqrt{4+5} = \sqrt{9} = 3$

$F_1(-5-3, 0) \Rightarrow F_1(-8, 0)$

$F_2(-5+3, 0) \Rightarrow F_2(-2, 0)$

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Question 4) Foci of hyperbola are $(3,-6)$ and $(3,10)$ and $K=5$

(1) what does K mean?
(2) Find the equation?
(3) Roughly sketch?

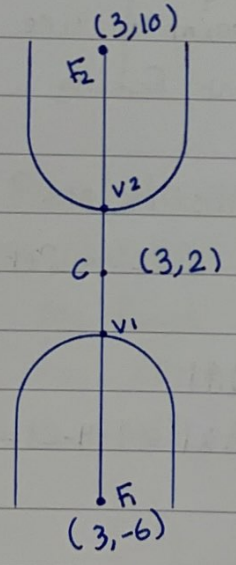
answer: if choose a point on the figure say P, the substraction of the distances to both the Focus points (Foci) from the point P is equal to a constant amount and that's K.

$$\big|\,|PF_1| - |PF_2|\,\big| = K$$

Take the absolute value to avoid negative signs becaus $K>0$

$F_1(3,-6)$ & $F_2(3,10)$

↳ They lie on a vertical line ⇒ The major axis



* Find c by finding the average of The $\overline{F_1F_2}$

$X_0 = 3 \rightarrow$ because it's on a vertical

$Y_0 \Rightarrow$ average $\Rightarrow \frac{10-6}{2} = 2$

* Finding The vertices by $K/2$

$K=5 \qquad K/2 = 2.5$

$V_1(3, 2-2.5) \Rightarrow V_1(3, -0.5)$

$V_2(3, 2+2.5) \Rightarrow V_2(3, 4.5)$

* To write the equation we need b^2

$|CF_2| = \sqrt{\left(\frac{K}{2}\right)^2 + b^2}$

↓

$|\Delta Y| = |10-2| = 8$

$8^2 = \left(\sqrt{6.25 + b^2}\right)^2$

$b^2 = 8^2 - 6.25 = 57.75$

The equation

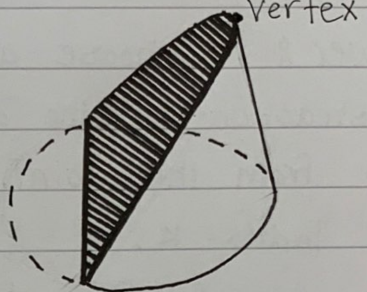
$$\frac{(x-3)^2}{6.25} - \frac{(y-2)^2}{57.75} = 1$$

* Solve The question with $K=6$

Parabola

a **parabola** is a plane curve that is symmetrical and is approximately u-shaped, it involves a focus point and a line facing its vertex.

A **parabola** is created from the intersection of a right circular conical surface and a plane.



when sketching Fix a point (F) on horizontal or vertical line
Find all points in the xy - plane
|point F| = |point line|

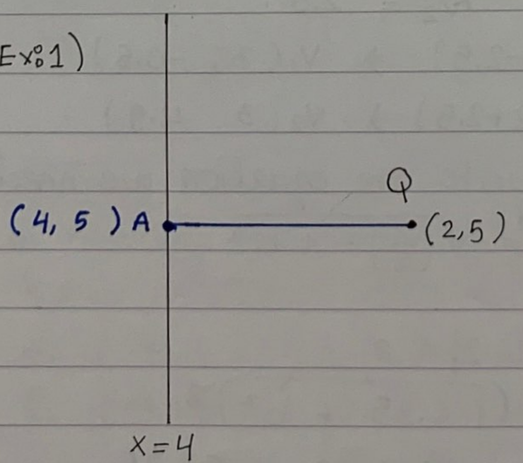
- Finding the distance between a point & a line

step 1) draw a perpendicular from the point to line

step 2) Find The coordinates of the point facing the first one

step3) The distance between the two points is the distance between the line and the point $|P_1L| = |P_2P_1|$

Ex:1)



$|QL| = |QA|$

$|QA| = |\Delta x| = |-4-2| = 6$

$|QL| = 6$

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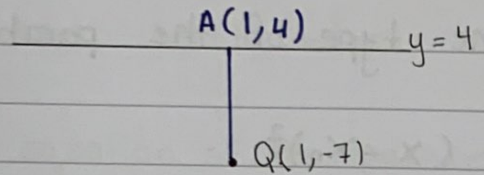
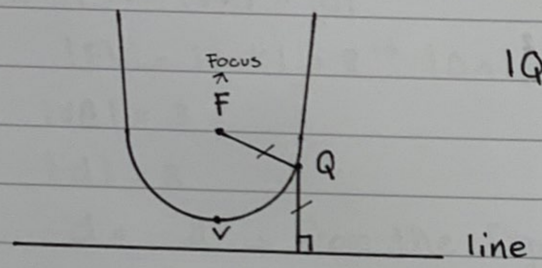
Ex: 2)



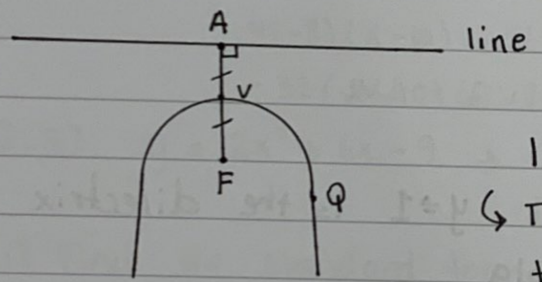
$|QL| = |QA| = |\Delta y|$

$|\Delta y| = |4-(-7)| = 11$

a **Parabola** is a set of points, and they satisfy the condition | Point F | = | Point line |



$|QF| = |QL|$



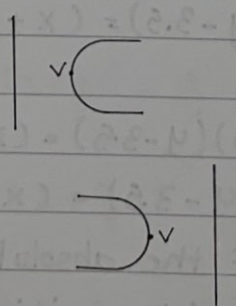
$|QF| = |QL|$

$|FV| = |VA| = |VL| = |d|$

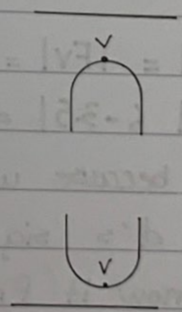
↳ The vertex is a point on the figure as well

a **Parabola** involves a point, the focus F & a line the directrix

* the focus does not lie on the directrix
* the directrix is facing the vertex
* The left-right parabola



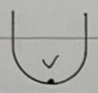
* The up-down parabola



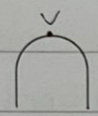
To determine the type of the parabola

(1) $4d(y-y_0) = (x-x_0)^2$

$(x_0, y_0) \rightarrow$ vertex

IF $d > 0 \rightarrow$ 

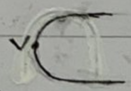
$d \rightarrow$ positive

IF $d < 0 \rightarrow$ 

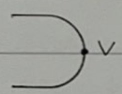
$d \rightarrow$ negative

(2) $4d(x-x_0) = (y-y_0)^2$

$(x_0, y_0) \rightarrow$ vertex

IF $d > 0 \rightarrow$ 

$d \rightarrow$ positive

IF $d < 0 \rightarrow$ 

$d \rightarrow$ negative

note that: $|d| = |FV| = |VA| = |VL|$

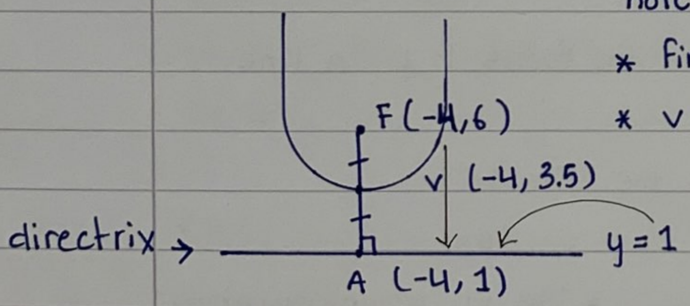
Question 1) $F \Rightarrow (-4, 6)$ $y = 1$ is the directrix

↳ draw such parabola

note: v is the midpoint of the line segment

* find the average of y to find v

* $v(-4, y) \rightarrow 6+1/2 \Rightarrow 3.5$



The equation:

$4d(y-y_0) = (x-x_0)^2$

$4d(y-3.5) = (x+4)^2$

$|d| = |FV| = |FA| =$

$|6-3.5| = 2.5$

↳ because we don't know d's sign (−/+) because of the absolute value

→ we know it from the shap U → d is positive

* $4(2.5)(y-3.5) = (x+4)^2$

* $10(y-3.5) = (x+4)^2$

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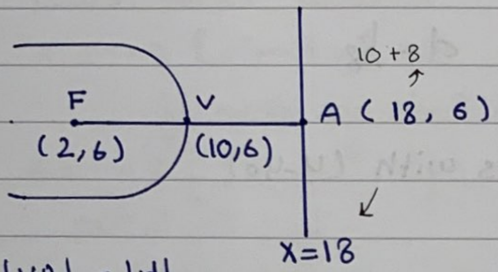
Q:2) F → (2,6) V = (10, 6)

(1) sketch

(2) Find the equation of the parabola



* by the coordinates of V & F, we can tell they lie on a horizontal line and we can conclude how the parabola will look like

* $|FV| = |VA| = |d|$

$|FV| = |2-10| = 8 \rightarrow |\Delta x|$

$|VA| = 8$

$|d| = 8$

$d = -8$ → From the figure

equation $4d(x-x_0) = (y-y_0)^2$

$4(-8)(x-10) = (y-6)^2$

$-32(x-10) = (y-6)^2$

Q:3) $y = 2x^2 + 6x - 9$ → A parabola that's not written in the standard form

(1) Find the standard form of the given parabola

(2) Find focus, vertex & directrix

* $y = 2x^2 + 6x - 9$

make coefficient of $x^2 = 1$.

$y = 2[x^2 + 3x] + 9$

$y = 2[[(x+\frac{3}{2})^2 - \frac{9}{4}]] - 9$

half coefficient of x ↵ ↳ when squaring $x+\frac{3}{2}$ this the result $x^2 + 3x + \frac{9}{4}$

$y = 2(x+\frac{3}{2})^2 - \frac{18}{4} - 9$

$y = 2(x+\frac{3}{2})^2 - \frac{27}{4}$

$(y+\frac{27}{4}) = 2(x+\frac{3}{2})^2$ ÷ 2 ⇒ $\frac{1}{2}(y+\frac{27}{4}) = (x+\frac{3}{2})^2$

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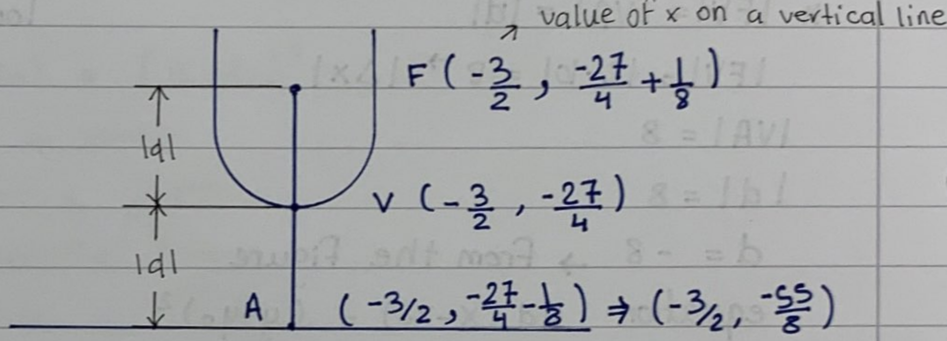
$\frac{1}{2}\left(y+\frac{27}{4}\right)=\left(x+\frac{3}{2}\right)^2 \rightarrow$ standard form

Vertex $=\left(-\frac{3}{2}, -\frac{27}{4}\right)$

$4d=\frac{1}{2} \Rightarrow d=\frac{1}{8}$

* d is positive
* equation starts with $(y-y_0)$
* The figure :



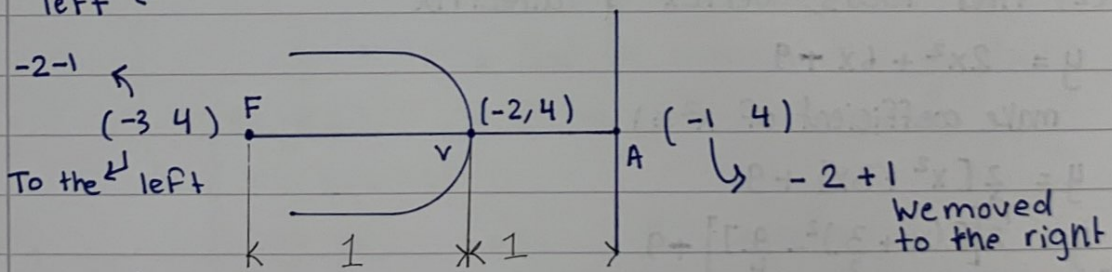
$y=\frac{-55}{8}$

Q:4) $-8(x+2)=\underline{2}(y-4)^2$ $\div 2$

1) sketch
2) Find focus, vertex, directrix

$-4(x+2)=(y-4)^2$ $V, (x_0, y_0)$

left



* $4d=-4$

$d=-1$

$|d|=1$

$1=|FV|=|FA|$

$x=-1$

$\therefore$ directrix $=-1$

Q5) $x = -2y^2 + 8y - 15$

(1) write in a standard

(2) sketch

(3) find focus, vertex, directrix

Solution: $x = -2\,[y^2 - 4y] - 15$

↳ make the coefficient of $x^2 = 1$

$x = -2\left[\,[(y-2)^2 - 4]\,\right] - 15$

half of coff. x

$x = -2(y-2)^2 + 8 - 15$

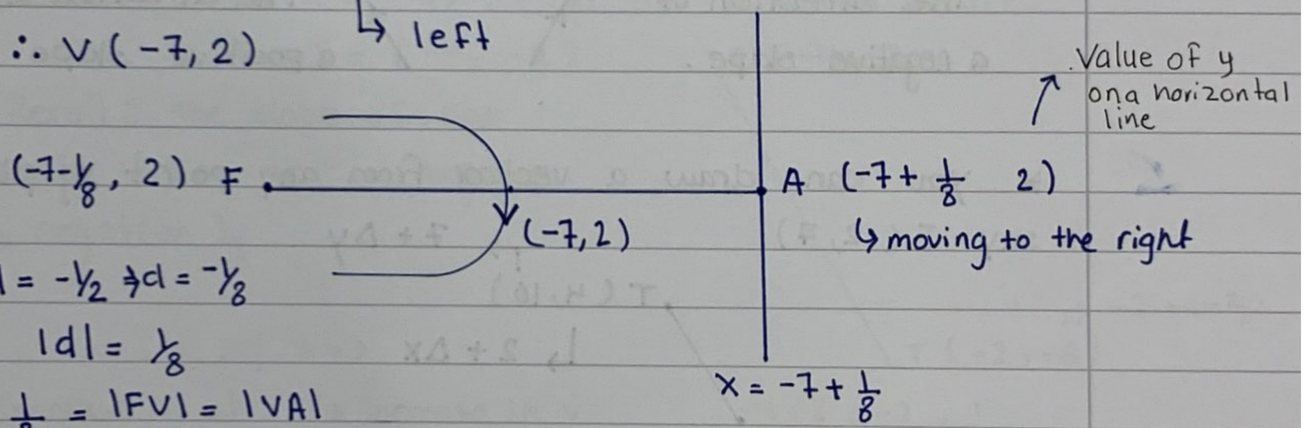
$x = -2(y-2)^2 - 7$

$(x+7) = -2(y-2)^2 \quad \div 2$

Final equation: $-\frac{1}{2}(x+7) = (y-2)^2$

↳ left

$\therefore V(-7, 2)$



$4d = -\frac{1}{2} \Rightarrow d = -\frac{1}{8}$

$|d| = \frac{1}{8}$

$\frac{1}{8} = |FV| = |VA|$

Intro. about Vectors in 2D

Definition → a vector is a directed line segment connects between two points

a line segment → has an initial and a terminal point

notations that present a vector:

$v = \langle 2, 3 \rangle \neq (2, 3)$

$v = 2i, 3j$

a vector can be drawn from any point in the xy-plane